Cross-helicity dynamo effect in magnetohydrodynamic turbulent channel flow

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A large eddy simulation of magnetohydrodynamic (MHD) turbulent channel flow is carried out to investigate the dynamo mechanism. It is shown that the streamwise component of the mean magnetic field is generated and sustained due to the effect of the turbulent electromotive force. The Reynolds-averaged turbulence model for MHD flows is assessed; it is suggested that the cross-helicity averaged turbulence model contributes to the turbulent electromotive force; that is, the electromotive force parallel to the mean vorticity is generated due to the turbulent cross helicity. To verify the importance of the cross-helicity dynamo, the transport equation for the turbulent electromotive force is evaluated; it is confirmed that the term involving the cross helicity and the mean vorticity is the main production term for the turbulent electromotive force. The transport equations for the turbulent kinetic and magnetic energies are also examined to discuss the dynamo mechanism from the viewpoint of the energy transfer. © 2010 American Institute of Physics.

I. INTRODUCTION

The behavior of magnetic fields in astro-/geophysical objects and in plasma controlled fusion devices is closely related to the motion of electrically conducting fluids such as the plasma gas and the molten iron. In magnetohydrodynamic (MHD) flows at high magnetic Reynolds numbers, turbulent motions enhance the diffusion of the magnetic field like the scalar transport in non-MHD turbulent flows; the effective diffusivity due to turbulence is much greater than the molecular diffusivity. However, the turbulent diffusion effect cannot account for large-scale magnetic fields observed in actual MHD flows. It is expected that some dynamo action exists, which generates and sustains the large-scale magnetic field against the diffusion effect. For example, the $\alpha$ dynamo is the most famous dynamo mechanism. Helical fluid motions induce the electromotive force in the direction of the mean magnetic field. The cross-helicity dynamo was also proposed and investigated. The electromotive force is generated in the direction of the mean vorticity when the cross helicity, the correlation between the velocity and the magnetic field, has a nonzero value.

In the induction equation for the mean magnetic field, the turbulent electromotive force appears as an unknown term. In order to close the magnetic-field equation, the term needs to be modeled using the mean field. The turbulent electromotive force involves both the turbulent diffusion effect and the turbulent dynamo effect. The mean-field dynamo theory has been studied theoretically and numerically; for example, the dynamo mechanism for the solar magnetic field is investigated in detail. On the other hand, a statistical theory called the two-scale direct-interaction approximation (TSDIA) was applied to MHD turbulence; it was originally developed for non-MHD flows and was extended to MHD flows. Using this theory, several turbulence models were proposed to investigate the magnetic field in astro-/geophysical objects and in plasma controlled fusion devices. However, these models are not fully assessed using experiment and numerical simulations in contrast to turbulence models for non-MHD flows.

The direct numerical simulation (DNS) and the large eddy simulation (LES) are expected to be useful for validating the turbulence models and the mean-field dynamo theory. For example, several three-dimensional simulations for solar dynamo have been carried out. The simulations for solar dynamo need to involve fairly complex physical phenomena such as the system rotation, the thermal convection, and the fluid compressibility. As a first step to understand the dynamo mechanism, simulations of simple MHD flows must be helpful. Moreover, the DNS, which is a powerful tool for basic turbulence research, is limited to low- or moderate-Reynolds-number flows because a large number of grid points are required. To examine properties of high-Reynolds-number flows, the LES of a simple flow is an important approach as already done for non-MHD turbulence research.

Several subgrid-scale (SGS) models for the LES of MHD turbulence were proposed. Using the TSDIA, Yoshizawa developed a SGS model for MHD turbulence. Hamba modified the SGS model so that it involves the $\alpha$ dynamo effect. Zhou and Vahala also theoretically derived a SGS model taking into account the dynamo theory. Several dynamic SGS models have been tested for the LES of a decaying isotropic MHD turbulence. Miki and Menon proposed a localized dynamic SGS model for MHD turbulence. Chernyshov et al. carried out a LES of a heat conducting compressible MHD turbulence. Compared to the decaying isotropic turbulence, the number of LES of inhomogeneous MHD turbulence is small. Theobald et al.
carried out a LES of two-dimensional rectangular box for MHD turbulence considering the outer layer of the Sun. Matsushima examined a scale-similarity SGS model in a small rectangular region of the Earth’s core. To our knowledge, no LES of turbulent channel flow at high magnetic Reynolds number has been done to investigate statistical quantities of MHD turbulence in detail. On the other hand, several LES of turbulent channel flow at low magnetic Reynolds number have been carried out. These simulations did not treat the turbulent electromotive force due to the approximation of the low magnetic Reynolds number. However, detailed turbulent statistics reveal the properties of the turbulent channel flow. Therefore, a LES of turbulent channel flow at high magnetic Reynolds number must be useful for examining the dynamo mechanism. A simulation of MHD channel flow with a magnetic Prandtl number equal to unity does not directly correspond to a specific laboratory flow. A MHD flow in a circular pipe is often numerically studied as an approximation to torus devices of plasma controlled fusion such as the Tokamak and the reversed field pinch. Since a channel flow can be simulated more accurately than a pipe flow, the former is chosen as a basic flow for the dynamo study.

In this work, we carried out a LES of turbulent channel flow of an electrically conducting fluid to obtain a statistically steady state with a large-scale magnetic field. We examine the mean field and the turbulence statistics to investigate the dynamo mechanism of the MHD turbulence. This paper is organized as follows. In Sec. II we describe the model equations and the numerical method for the LES of MHD turbulent flow. In Sec. III we evaluate the turbulence statistics such as the turbulent electromotive force and its transport equation to investigate the dynamo effect. We also examine the energy transfer with respect to the turbulent kinetic and magnetic energies to clarify the properties of the turbulent channel flow. Concluding remarks are given in Sec. IV.

II. MODEL EQUATIONS AND NUMERICAL METHOD

In this paper, we adopt Alfvén velocity units and replace the magnetic field \( \mathbf{b} / \sqrt{\rho \mu_0} \rightarrow \mathbf{b} \), the electric current density \( j / \sqrt{\rho \mu_0} \rightarrow j \), and the electric field \( e / \sqrt{\rho \mu_0} \rightarrow e \), where \( \rho \) is the fluid density and \( \mu_0 \) is the magnetic permeability. In the LES, we treat the grid-scale (GS) velocity \( \mathbf{u} \) and the GS magnetic field \( \mathbf{b} \). The overbar denotes filtering defined by

\[
\bar{f}(x) = \int \int \int d\mathbf{x}' G(x - x', \Delta) f(x'),
\]

where \( G \) is the filter function and \( \Delta \) is the filter width. The Navier–Stokes and continuity equations for the GS velocity and the induction equation for the GS magnetic field are given by

\[
\frac{\partial \mathbf{u}}{\partial t} = - \nabla \cdot (\mathbf{u} \mathbf{u} - \mathbf{b} \mathbf{b} + \mathbf{e}) - \nabla p_M + \nu \nabla^2 \mathbf{u},
\]

\[
\nabla \cdot \mathbf{b} = 0,
\]

\[
\frac{\partial \mathbf{b}}{\partial t} = - \nabla \times \mathbf{e},
\]

\[
\mathbf{e} = - \mathbf{u} \times \mathbf{b} - \mathbf{e}_M + \lambda_M \mathbf{j},
\]

\[
\mathbf{v} \cdot \mathbf{b} = 0,\]

where \( \nu \) is the viscosity, \( \lambda_M \) is the magnetic diffusivity, \( p_M(=\bar{p} + \bar{b}^2/2 + \tau) \) is the GS total pressure, \( \mathbf{j} = \nabla \times \mathbf{b} \) is the GS current density, \( \tau \) is the deviatoric part of the SGS stress, and summation convention is used for repeated indices. The SGS stress \( \tau \) and the SGS electromotive force \( \mathbf{e}_M \) are defined as

\[
\tau_{ij} = u_i u_j - \bar{u}_i \bar{u}_j - (u_i u_j - \bar{u}_i \bar{u}_j),
\]

\[
\mathbf{e}_M = \mathbf{u} \times \mathbf{b} - \bar{\mathbf{u}} \times \bar{\mathbf{b}},
\]

respectively. In order to close the system of Eqs. (2)–(5), we need to model the above SGS terms; we adopt eddy-viscosity and eddy-diffusivity models such as the Smagorinsky model for non-MHD turbulence. The SGS stress and electromotive force are expressed as

\[
\tau_{ij} = - \nu_{SGS} \bar{s}_{ij},
\]

\[
\mathbf{e}_M = - \lambda_{SGS} \mathbf{j},
\]

respectively, where the SGS viscosity and diffusivity are given by

\[
\nu_{SGS} = C_\nu \Delta^2 \left( \frac{1}{2} C_p \bar{\rho}_{ij} + C_m \bar{B}_{ij}^2 \right)^{1/2},
\]

\[
\lambda_{SGS} = C_\lambda \Delta^2 \left( \frac{1}{2} C_p \bar{\rho}_{ij} + C_m \bar{B}_{ij}^2 \right)^{1/2}.
\]

Here, the filter width is obtained from \( \Delta = (\Delta_x \Delta_y \Delta_z)^{1/3} \), where \( \Delta_x \), \( \Delta_y \), and \( \Delta_z \) are the filter width in the \( x \), \( y \), and \( z \) directions, respectively; the model constants are given by

\[
C_\nu = (5/7) C_p, \quad C_\lambda = 0.046.
\]

This SGS model was derived by extending the Smagorinsky model for non-MHD turbulence to MHD. It was assumed that the production and dissipation terms in the transport equation for the SGS energy are balanced to each other. The values of model constants were determined, considering the theoretical value of the turbulent magnetic Prandtl number and the correspondence to the Smagorinsky model in the non-MHD limit. The detailed explanation of the SGS model is given in the Appendix.

Using the above equations, we carried out a LES of MHD channel flow driven by a constant pressure gradient \( -\partial p_0 / \partial z \). The computational domain is \( L_x \times L_y \times L_z = 8 \pi \times 2 \times 2 \pi \) and the number of grid points is \( N_x \times N_y \times N_z = 256 \times 64 \times 64 \) where \( x \), \( y \), and \( z \) denote the streamwise, wall-normal, and spanwise directions, respectively, as shown in Fig. 1. The length scale is normalized by the channel half width \( L_y/2 \), whereas the velocity and the magnetic field are normalized by the friction velocity \( u_* = (\sqrt{\partial p_0 / \partial y \mathbf{|}_{wall}}) \), where \( \langle u_* \rangle \) is the ensemble-averaged velocity. For a statistically steady channel flow, this normalization leads to...
The Reynolds and magnetic Reynolds numbers based on the friction velocity are set to \( \text{Re}_f = u_f (L_x/2)/v = 395 \) and \( \text{Re}_m = u_f (L_x/2)/l_M = 395 \), respectively. The magnetic Prandtl number \( v/\kappa_M \) is set equal to unity as a first step of numerical analysis. The effect of high or low magnetic Prandtl numbers on the numerical results is an interesting issue and needs to be examined in the future.

The periodic boundary conditions are imposed in the \( x \) and \( z \) directions. The nonslip condition \( \mathbf{u} = 0 \) is imposed at the wall at \( y = \pm 1 \). The wall is treated as an insulator; that is, the magnetic field at the wall is given by

\[
\mathbf{b} = \mathbf{b}_{\text{ins}},
\]

where \( \mathbf{b}_{\text{ins}} \) is the magnetic field in the insulator which is determined by the potential \( \phi \) as follows:

\[
\mathbf{b}_{\text{ins}} = -\nabla \phi, \quad \nabla^2 \phi = 0.
\]

Like the LES of non-MHD channel flow, the van Driest damping function is applied in the near-wall region by replacing the filter width as

\[
\Delta \rightarrow \Delta f_r, \quad f_r = 1 - \exp(-y^*/A^*). \tag{16}
\]

Here, \( A^* = 25 \) and \( y^* = u y_\tau / v \), where \( y_\tau \) is the distance from the wall.

A statistically steady velocity field of the non-MHD turbulent channel flow is used as the initial condition for the velocity. A random seed field with zero mean is used as the initial condition for the magnetic field. Unlike the Hartmann flow, the wall-normal component of the mean magnetic field \( B_z \) is set to zero. This is because we want to realize a turbulent field without the mean-field electromotive force \((\mathbf{U} \times \mathbf{B})_z\), which induces the mean magnetic field \( B_z \) in a trivial manner so that we can observe whether the turbulent field generates the mean magnetic field or not. We use the second-order finite-difference scheme in space and the Adams–Bashforth method for time marching. The time step is set to \( \Delta t = 5 \times 10^{-4} \). The computation was run for a sufficient length of time to obtain a nearly statistically steady turbulent field.

\[
- \frac{\partial p}{\partial x} = u_t^2 = 1. \tag{13}
\]

III. RESULTS AND DISCUSSION

A. Time evolution of turbulent field

First, we examine the time evolution of the velocity and the magnetic field. Figure 2 shows the time history of the volume-averaged velocity and magnetic field given by

\[
U_{\text{vol}} = \frac{1}{L_y} \int (\mathbf{u}_x)_z \, dy \quad \text{and} \quad B_{\text{vol}} = \frac{1}{L_y} \int (\mathbf{b}_x)_z \, dy,
\]

respectively, where \( \langle \cdot \rangle_z \) denotes the \( x-z \) plane average. The former and latter integrals represent the flow rate and the magnetic flux in the \( x \) direction, respectively. Since the pressure gradient \( -\partial p/\partial x \) is applied, the mean velocity is driven in the positive \( x \) direction. The initial value of the mean velocity is \( U_{\text{vol}} = 17 \), which corresponds to that of the non-MHD channel flow. Showing an overshoot, the mean velocity rapidly increases to about \( U_{\text{vol}} = 24 \) until \( t = 100 \), where time is normalized by \( (L_y/2)/u_f \). This rapid increase is because the initial non-MHD velocity field is adjusted to the MHD one. The mean velocity then increases gradually to about \( U_{\text{vol}} = 27 \) until \( t = 600 \) and reaches a nearly steady state.

The mean magnetic field also shows positive values. It gradually increases to about \( B_{\text{vol}} = 3.7 \) until \( t = 400 \) and reaches a nearly steady state. In contrast to the mean velocity driven by the pressure gradient, the reason for the positive mean magnetic field is not trivial because there is no mean-

![FIG. 1. Computational domain of the channel flow.](image-url)
field electromotive force as mentioned before. In fact, in some other runs the mean magnetic field becomes negative, although its magnitude is nearly the same as that in the present run (not shown here). Therefore, some mechanism exists, which induces nonzero magnetic flux in the x direction. It takes a fairly long time to reach a steady state compared to the non-MHD channel flow, which requires only about \( t=10 \) in usual cases.

Figure 3 shows the time history of the volume-averaged turbulent kinetic and magnetic energies given by

\[
\frac{1}{L_y} \int (1/2) (\overline{u'_i}^2) z \, dy \quad \text{and} \quad \frac{1}{L_y} \int (1/2) (\overline{b'_i}^2) z \, dy, \quad \text{where } f' = f - \langle f \rangle. \]

In the time period until \( t=400 \), the kinetic and magnetic energies exhibit nearly the same value although they fluctuate in time. After \( t=400 \) when the mean magnetic field reaches a steady state in Fig. 2(b), the kinetic energy is greater than the magnetic energy. This behavior suggests that the kinetic and magnetic energies are equipartitioned in the case of weak mean magnetic field, whereas the former becomes greater than the latter due to the strong mean magnetic field. Figures 2 and 3 clearly show that a nearly steady state for volume-averaged quantities is achieved at \( t=600 \). We examine the turbulent field in the steady state in Secs. III B–III D.

Here, we mention the reason why a LES is carried out rather than a DNS. It is clear that DNS is more desirable because the results are unaffected by the SGS modeling. A well resolved DNS can be conducted either by decreasing the Reynolds number or by increasing the grid resolution. First, it was shown in preliminary calculations that turbulence decays and a statistical steady state is not obtained for smaller Reynolds number such as \( Re_x = Re_m = 180 \). The Reynolds number as high as the present value seems necessary. Second, in the present LES the grid number is not very large, but the computing time is much longer than that for non-MHD channel flow. Since plane-averaged quantities fluctuate in time as shown in Fig. 3, a long time average is required to obtain reliable statistics. At present, it is difficult to carry out a DNS of this MHD flow for such a long time. It needs to be done in near future to verify the present results and to assess the SGS model.

B. Turbulent statistics

Hereafter, statistical quantities are obtained by averaging over the \( x-z \) plane and in time from \( t=600 \) to 1600. This average is denoted by \( \langle \cdot \rangle \) and physical quantities are decomposed as \( f = (f) + f' \). Some mean values are also denoted by upper-case letters such as \( U = \langle \textbf{u} \rangle \) and \( B = \langle \textbf{b} \rangle \). We examine the profiles of statistical quantities as functions of \( y \).

Figure 4(a) shows the mean velocity profiles \( U_x \). The solid line represents the velocity of the present simulation and the dashed line denotes the velocity of the LES of non-MHD channel flow with the same code; the latter velocity is adopted as the initial velocity field of the present simulation for the MHD flow. The dotted line is the result of the DNS of non-MHD channel flow by Moser et al. The mean velocity in the non-MHD case obtained from DNS of Moser et al. (Ref. 31) is also plotted.
the velocity gradient is steep near the wall becomes broader compared to the non-MHD case. For example, the velocity gradient at \( y = \pm 0.8 \) in the MHD case is much greater than that in the non-MHD case.

Figure 4(b) shows the profile of the mean magnetic field \( B_x \). Although its magnitude is about 10% of the mean velocity, it clearly shows a nonzero profile with a maximum at the channel center. Therefore, a large-scale magnetic field is generated and sustained; some dynamo effect exists in this MHD channel flow. Because of the insulating boundary condition at the wall, the magnetic field is close to zero at the wall. Like the mean velocity profile, the steep gradient of the mean magnetic field is seen near the wall.

Figure 5 shows profiles of the streamwise, wall-normal, and spanwise components of the turbulent intensities \( \sqrt{\langle u_x'^2 \rangle} \), \( \sqrt{\langle u_y'^2 \rangle} \), \( \sqrt{\langle b_x'^2 \rangle} \), \( \sqrt{\langle b_y'^2 \rangle} \), and \( \sqrt{\langle b_z'^2 \rangle} \). The velocity intensity in the non-MHD case is also plotted. In Fig. 5(a) the streamwise velocity fluctuation in the MHD case is twice as large as that in the non-MHD case. It is suggested that the large kinetic energy in the MHD case is produced because of the large mean velocity shown in Fig. 4(a). As already mentioned for its time evolution in Fig. 3, the magnetic-field fluctuation is less than the velocity fluctuation. The velocity and magnetic-field intensities show similar profiles; they have a peak near the wall and gradually decrease toward the channel center.

On the other hand, the wall-normal velocity fluctuation in the MHD case is much less than that in the non-MHD case, as shown in Fig. 5(b). The magnetic-field fluctuation is as small as the velocity fluctuation in the MHD case. The same holds for the spanwise components plotted in Fig. 5(c). Therefore, the anisotropy of the turbulent kinetic energy in the MHD case is much greater than that in the non-MHD case. This strong anisotropy suggests that the effect of the energy redistribution between the three components of the kinetic energy is weaker in the MHD case.

So far we have shown that the mean velocity in the MHD case is greater than that in the non-MHD case and that the positive mean magnetic field is generated. Now, in order to clarify their mechanism, we examine the transport equations for the mean velocity and the mean magnetic field. First, to understand the reason for the increase in the mean velocity, we examine the mean velocity equation, or the balance of the mean momentum given by

\[
\frac{\partial U_i}{\partial t} = -\frac{\partial}{\partial y} \left( \langle u_i' u_j' \rangle - \langle b_j' B_i' \rangle + \langle \tau_{ij} \rangle - \nu \frac{\partial U_i}{\partial y} \right) + 1. \tag{17}
\]

When the turbulent field is statistically steady, the time derivative on the left-hand side vanishes. Integrating each term in the \( y \) direction from the wall at \( y = -1 \) with the boundary condition \( \nu U_i / \partial y = 1 \), we have

\[
\langle u_i' u_j' \rangle - \langle b_j' B_i' \rangle + \langle \tau_{ij} \rangle - \nu \frac{\partial U_i}{\partial y} = y. \tag{18}
\]

Figure 6 shows profiles of the three terms on the left-hand side of Eq. (18): the GS, SGS, and viscous terms. The total of the three terms and the GS stress term in the non-MHD case are also plotted. The total value is nearly equal to \( y \); this linear profile shows that a statistically steady state is achieved in the present simulation. The GS stress \( \langle u_i' u_j' \rangle - \langle b_j' B_i' \rangle \) is small near the wall and is dominant away from the wall. Its magnitude is smaller than that in the non-MHD case. This small magnitude is because the wall-normal velocity fluctuation \( \langle u_y'^2 \rangle \) is smaller in the MHD case, as shown
in Fig. 5(b). The SGS stress $\langle \tau_{xy} \rangle$ is much less than the GS stress and is comparable with the viscous term at most. Considering the small SGS stress, we expect that the SGS modeling does not affect the result very much. On the other hand, the viscous term $-\nu \partial U_y/\partial y$ is dominant near the wall and rapidly decreases away from the wall. From the above momentum balance, we can explain the reason for the large mean velocity in the MHD case as follows. The GS stress decreases in the MHD case because of the small wall-normal velocity fluctuation. The total of the three terms should be equal to $y$ in both the MHD and non-MHD cases and the SGS term is irrelevant. Therefore, the viscous term proportional to the velocity gradient must increase in the MHD case, leading to the increase in the mean velocity, as shown in Fig. 4(a).

The reason for the large mean velocity can also be explained by considering the eddy viscosity model for the Reynolds-averaged Navier–Stokes (RANS) equation. The shear stress can be expressed by

$$\langle u'' u''_y - b'' b''_y \rangle = - \nu_{t} \frac{\partial U_y}{\partial y}. \quad (19)$$

The eddy viscosity $\nu_{t}$ is expected to be proportional to the wall-normal stress $\langle u'' u''_y \rangle$ in shear turbulence.\textsuperscript{32} The shear stress in the MHD case is about 70\% of that in the non-MHD case, whereas the wall-normal stress in the MHD case is about one-quarter of that in the non-MHD case, as shown in Fig. 5(b). Therefore, the velocity gradient must increase in the MHD case compared to the non-MHD case.

Next we investigate the mechanism of the sustainment of the mean magnetic field shown in Fig. 4(b). The induction equation for the mean magnetic field and the balance of the electromotive force are given by

$$\frac{\partial B_x}{\partial t} = - \frac{\partial \langle \epsilon_z \rangle}{\partial y}, \quad (20)$$

$$-\langle \epsilon_z \rangle = (U \times B)_z + (\bar{\mathbf{r}} \times \bar{\mathbf{B}})_z + \langle \epsilon_{M} \rangle - \lambda_M J_z, \quad (21)$$

respectively, where $\langle \bar{\mathbf{r}} \times \bar{\mathbf{B}} \rangle = \langle \bar{u}_x \bar{B}_y - \bar{u}_y \bar{B}_x \rangle$ and $\mathbf{J} = \nabla \times \mathbf{B}$. Since the initial condition $\int \bar{B}_x dx dz = 0$ was set and its time derivative $\int \partial \bar{B}_x / \partial t dx dz = \int (\nabla \times \mathbf{E}) dx dz$ always vanishes due to the periodic boundary condition for $\bar{E}$ in the $x$ and $z$ directions, the wall-normal mean magnetic field $B_x$ vanishes. The wall-normal mean velocity $U_y$ is also zero due to the incompressible condition. Hence, the mean-field electromotive force $(U \times B)_z = U_x B_z - U_z B_x$ vanishes. Figure 7 shows profiles of the remaining three terms on the right-hand side of Eq. (21): the GS, SGS, molecular diffusion (Ohmic) terms. The total value $-\langle \epsilon_z \rangle$ and the two parts of the GS term, $\langle \bar{u}_x \bar{B}_y \rangle$ and $-\langle \bar{u}_y \bar{B}_x \rangle$, are also plotted. Since a very long time average is taken and a statistically steady state is achieved, the total electric field $\langle \epsilon_z \rangle$ is nearly zero. At $-0.9 < y < 0.9$ the SGS term $\langle \epsilon_{M} \rangle$ and the Ohmic term $-\lambda_M J_z$ show negative gradient. The negative gradient means the diffusion effect because it gives a negative value of $\partial B_x / \partial t$ in Eq. (20), leading to the decrease in the mean magnetic field $B_x$. On the other hand, the GS term $\langle \bar{u}_x \bar{B}_y \rangle - \langle \bar{u}_y \bar{B}_x \rangle$ shows positive gradient at $-0.9 < y < 0.9$. This positive gradient represents the turbulent dynamo effect; that is, turbulent motion generates the mean magnetic field against the diffusion effect. The GS term consists of two parts; they have opposite signs and their magnitudes are large. The magnitude of $\langle \bar{u}_x \bar{B}_y \rangle$ is slightly greater than that of $-\langle \bar{u}_y \bar{B}_x \rangle$; this difference leads to the positive gradient of the GS term.

In the present simulation, a positive mean magnetic field $B_x$ is obtained. If $b_z$ is a solution of the incompressible MHD equation, $-b_z$ is also a solution. In fact, some other runs in which a different random field is used for the initial condition give negative $B_x$. It is expected that the number of runs with positive $B_x$ should be statistically the same as the number of runs with negative $B_x$.\textsuperscript{31}
C. Dynamo modeling

In the preceding subsection, it was shown that the mean magnetic field is not generated by the mean-field electromotive force (U × B), but by the turbulent electromotive force \( \langle \mathbf{u} \times \mathbf{b} \rangle \). In the mean-field dynamo theory and in the RANS model, the turbulent electromotive force needs to be modeled in order to close the mean field equations. A model for the turbulent electromotive force is given by

\[
\langle \mathbf{u} \times \mathbf{b} \rangle = \alpha \mathbf{B}_x - \beta \mathbf{J}_x + \gamma \mathbf{J}_z,
\]

where \( \mathbf{J} = (\nabla \times \mathbf{U}) \) is the mean vorticity. The first, second, and third terms on the right-hand side represent the \( \alpha \) dynamo, the turbulent diffusivity, and the cross-helicity dynamo, respectively. It is known that the coefficients \( \alpha, \beta, \) and \( \gamma \) are related to turbulent statistics as follows. The \( \alpha \) dynamo is the most famous dynamo effect; it results from the helical motion of fluid and \( \alpha \) is proportional to the turbulent residual helicity \( -\mathbf{u} \cdot \nabla \times \mathbf{u} \). The turbulent diffusivity effect is always caused by the turbulent motion; \( \beta \) is proportional to the turbulent MHD energy \( (\mathbf{u}^2 + \mathbf{b}^2)/2 \). The cross-helicity dynamo is a recently proposed dynamo effect; \( \gamma \) is expected to be proportional to the turbulent cross helicity \( \langle \mathbf{u} \cdot \mathbf{b} \rangle \).

In order to clarify which dynamo action occurs against the turbulent diffusivity effect in the present MHD flow, we examine the terms on the right-hand side of Eq. (22) using the GS quantities. The \( \alpha \) dynamo term is proportional to the spanwise mean magnetic field \( B_z \) and the turbulent residual helicity \( -\langle \mathbf{u} \cdot \nabla \times \mathbf{u} \rangle \). The both quantities are very small in the present simulation (not shown here). On the other hand, the cross-helicity dynamo term can be estimated as follows. The GS cross helicity \( \langle \mathbf{u} \cdot \mathbf{b} \rangle \) and its three parts \( \langle \mathbf{u} \cdot \mathbf{b} \rangle, \langle \mathbf{u} \cdot \mathbf{b} \rangle, \) and \( \langle \mathbf{u} \cdot \mathbf{b} \rangle \) are plotted in Fig. 8, where the latter two terms are magnified ten times. The GS cross helicity shows positive values except for the near-wall region. Among the three parts the streamwise component \( \langle \mathbf{u} \cdot \mathbf{b} \rangle \) is dominant; the remaining two components exhibit small positive values. Figure 9 shows profiles of the mean vorticity \( \Omega = -\partial \mathbf{u} / \partial y \).

It takes negative (positive) values at \( -1 < y < 0 \) (\( 0 < y < 1 \)); its gradient is positive in the whole region. Considering Figs. 8 and 9, we can see that the cross-helicity dynamo term is expected to have a positive gradient except for the near-wall region. This positive gradient can account for the GS electromagnetic force appearing in Eq. (21). This result suggests that not the \( \alpha \) dynamo but the cross-helicity dynamo contributes to the turbulent electromotive force in the present MHD flow.

The above analysis shows that the cross-helicity dynamo can be an important candidate for the dynamo action. However, it is still possible that other dynamo mechanisms we did not treat are more effective than the cross-helicity dynamo. To verify that the cross-helicity dynamo actually works, we evaluate the transport equation for the turbulent electromotive force \( \langle \mathbf{u} \cdot \mathbf{b} \rangle \) given by

\[
\frac{\partial}{\partial t} \langle \mathbf{u} \cdot \mathbf{b} \rangle = P_E + P_E + P_{EB} + T_{EB} - e_E + D_E,
\]

\[
P_{EB} = -2\langle \mathbf{u} \cdot \mathbf{b} \rangle \frac{\partial U_x}{\partial y}, \quad P_E = \langle \mathbf{u}^2 + \mathbf{b}^2 \rangle \frac{\partial B_x}{\partial y},
\]

\[
P_{EB} = \left( \mathbf{u} \cdot \frac{\partial \mathbf{u}}{\partial x_k} - \mathbf{b} \cdot \frac{\partial \mathbf{b}}{\partial x_k} - \mathbf{u} \cdot \frac{\partial \mathbf{b}}{\partial x_k} + \mathbf{b} \cdot \frac{\partial \mathbf{u}}{\partial x_k} \right) B_k,
\]

where \( P_{EB}, P_{EM}, \) and \( P_{EB} \) are the production terms directly related to the mean velocity or the mean magnetic field. Detailed expressions for the triple correlation term \( T_{EB} \), the dissipation term \( e_E \), and the diffusion term \( D_E \) are omitted here. The first production term \( P_{EB} \) is closely related to the cross-helicity dynamo because it is proportional to the mean vorticity \( -\partial \mathbf{u} / \partial y \); the wall-normal component of the cross helicity \( \langle \mathbf{u} \cdot \mathbf{b} \rangle \) is also involved. The second production term \( P_{EM} \) corresponds to the turbulent diffusivity term because it is proportional to the mean magnetic-field gradient \( \partial \mathbf{B} / \partial y \). The third production term \( P_{EB} \) is related to the \( \alpha \) dynamo because it contains a part proportional to the mean magnetic field \( B_z \). We should note that the term \( P_{EB} \) also contains a part proportional to the streamwise magnetic field \( B_z \); this...
part represents the pumping dynamo effect which generates the electromotive force perpendicular to the magnetic field.\textsuperscript{3,33}

Figure 10 shows profiles of the six terms on the right-hand side of Eq. (23). The production term $P_{E,K}$ involving $\partial U_x/\partial y$ shows negative (positive) values at $-1 < y < 0$ ($y < -1$) like the turbulent electromotive force $\langle \overline{u_x^\prime b_y^\prime} - \overline{u_y^\prime b_x^\prime} \rangle$ itself. On the other hand, the production term $P_{E,B}$ involving $\partial B_x/\partial y$ shows opposite sign to $\langle \overline{u_x^\prime b_y^\prime} - \overline{u_y^\prime b_x^\prime} \rangle$, which represents the loss of the turbulent electromotive force. The production term $P_{E,B}$ involving $B_i$ also shows opposite sign to $\langle \overline{u_x^\prime b_y^\prime} - \overline{u_y^\prime b_x^\prime} \rangle$; the pumping dynamo part proportional to $B_i$ is dominant in this term, whereas the $\alpha$ dynamo part proportional to $B_i$ is negligible (not shown here). Therefore, this result clearly shows that the term involving the mean vorticity and the cross helicity actually produces the turbulent electromotive force; the cross-helicity dynamo contributes to the magnetic field generation in the present MHD channel flow.

The contributions of the SGS part were not discussed in the above analysis. This is because the SGS contributions are not directly related to the mean field. Since the SGS stress and electromotive force are expressed using the eddy viscosity and diffusivity, respectively, the SGS contributions are included in the dissipation term $\epsilon_{E,z}$ and the diffusion term $D_{E,z}$. For example, the dissipation term $\epsilon_{E,z}$ includes the following part:

$$
\epsilon_{\text{SGS}} = \left\langle \nu_{\text{SGS}} \left[ \frac{\partial \overline{u_x}}{\partial x} + \frac{\partial \overline{u_x}}{\partial y} \right] \frac{\partial \overline{b_y}}{\partial y} = \left( \frac{\partial \overline{u_x}}{\partial x} + \frac{\partial \overline{u_x}}{\partial y} \right) \frac{\partial \overline{b_y}}{\partial y} \right\rangle.
$$

(26)

This part is expected to decrease the magnitude of the turbulent electromotive force.

The mean vorticity $-\partial U_x/\partial y$ is created due to the pressure gradient $-\partial p_{\text{nl}}/\partial x$ with the nonslip boundary condition at the wall. However, the mechanism of producing the cross helicity is not clear. In order to understand the mechanism, we examine the transport equations for $\langle \overline{u_x^\prime b_y^\prime} \rangle$ and $\langle \overline{u_y^\prime b_x^\prime} \rangle$ given by

$$
\frac{\partial}{\partial t} \langle \overline{u_x^\prime b_y^\prime} \rangle = P_{WxK} + P_{WxM} + \Pi_{Wx} - \epsilon_{Wx} + D_{Wx},
$$

(27)

$$
P_{WxK} = - \langle \overline{b_y^\prime u_y^\prime} - \overline{b_x^\prime u_x^\prime} \rangle \frac{\partial U_k}{\partial y},
$$

(28)

$$
P_{WxM} = - \langle \overline{u_x^\prime u_y^\prime} - \overline{b_y^\prime b_x^\prime} \rangle \frac{\partial B_i}{\partial y},
$$

(29)

$$
\Pi_{Wx} = \left\langle \frac{\partial b_x^\prime}{\partial x} \right\rangle,
$$

(30)

Here, $P_{WxK}$ and $P_{WxM}$ are the production terms for $\langle \overline{u_x^\prime b_y^\prime} \rangle$ related to the mean field, whereas $\Pi_{Wx}$ and $\Pi_{Wy}$ are the redistribution terms that represent the transfer of the cross helicity between $\langle \overline{u_x^\prime b_y^\prime} \rangle$, $\langle \overline{u_y^\prime b_x^\prime} \rangle$, and $\langle \overline{u_y^\prime b_x^\prime} \rangle$. Detailed expressions for the dissipation terms $\epsilon_{Wx}$, $\epsilon_{Wy}$, and the diffusion terms $D_{Wx}$, $D_{Wy}$ are omitted here. The SGS contributions similar to Eq. (26) are included in the dissipation and diffusion terms. The same holds for the transport equations for the other quantities.

Figure 11(a) shows profiles of the five terms on the right-hand side of Eq. (27) for $\langle \overline{u_y^\prime b_x^\prime} \rangle$. The positive value of $\langle \overline{u_y^\prime b_x^\prime} \rangle$ apart from the wall is due mainly to the production term $P_{WxM}$ involving $\partial B_i/\partial y$. This production term consists of the shear stress $\langle \overline{u_x^\prime u_y^\prime} - \overline{b_y^\prime b_x^\prime} \rangle$ and the magnetic-field gradient $\partial B_i/\partial y$. The shear stress is created by the pressure gradient $-\partial p_{\text{nl}}/\partial x$ like the non-MHD channel flow; the stress always shows negative values at $-1 < y < 0$. The magnetic-field gradient depends on the magnetic field itself; the gradient is positive at $-1 < y < 0$ because the magnetic field is also positive at the channel center. Therefore, the production term $P_{WxM}$ given by Eq. (28) takes positive values at $-1 < y < 0$ because of the positive magnetic field $B_y$. On the other hand, the production term $P_{WxK}$ involving $\partial U_x/\partial y$ shows negative values. Its magnitude is large near the wall because of the steep velocity gradient. This is why the component $\langle \overline{u_x^\prime b_y^\prime} \rangle$ shows negative values near the wall in Fig. 8. The pressure-strain term also shows negative values although its magnitude is small. The negative value represents the transfer of the cross helicity from $\langle \overline{u_x^\prime b_y^\prime} \rangle$ to the other components. Figure 11(b) shows profiles of the three terms on the right-hand side of Eq. (30) for $\langle \overline{u_x^\prime b_y^\prime} \rangle$. Since there is no production term for $\langle \overline{u_x^\prime b_y^\prime} \rangle$, the pressure-strain term mainly generates the positive value of $\langle \overline{u_x^\prime b_y^\prime} \rangle$; that is, the cross helicity is transferred from $\langle \overline{u_x^\prime b_y^\prime} \rangle$ to $\langle \overline{u_y^\prime b_x^\prime} \rangle$ via $\Pi_{Wy}$. 
The mechanism for the sustainment of the mean magnetic field due to the cross-helicity dynamo is summarized in Fig. 12. The constant pressure gradient $-\partial p_0/\partial x$ creates the velocity gradient $\partial U_x/\partial y$ and the shear stress $\langle \tilde{u}_x^m \tilde{u}_y^m - \tilde{b}_x^m \tilde{b}_y^m \rangle$ like the non-MHD channel flow; the sign of these quantities does not depend on that of the mean magnetic field. If a positive magnetic field $B_x$ is once generated, the magnetic-field gradient and the shear stress lead to the positive value of the production term $P_{E,K}$ for $\langle \tilde{u}_x^m \tilde{b}_y^m \rangle$; positive cross helicity is generated. Due to the pressure-strain term $\Pi_{Wy}$, the cross helicity is transferred from $\langle \tilde{u}_x^m \tilde{b}_y^m \rangle$ to $\langle \tilde{u}_y^m \tilde{b}_y^m \rangle$. The component $\langle \tilde{u}_y^m \tilde{b}_y^m \rangle$ and the velocity gradient $\partial U_x/\partial y$ then lead to the negative (positive) production term $P_{E,K}$ at $-1 < y < 0$ ($0 < y < 1$) and the resulting electromotive force $\langle \tilde{u}_x^m \tilde{b}_y^m - \tilde{u}_y^m \tilde{b}_x^m \rangle$ is negative (positive); its gradient becomes positive. The positive gradient of $\langle \tilde{u}_y^m \tilde{b}_y^m - \tilde{u}_x^m \tilde{b}_x^m \rangle$ contributes to the increase in the positive magnetic field $B_x$ in the mean magnetic-field equation. Therefore, the positive magnetic field is sustained.

**D. Energy transfer**

The energy transfer is an important aspect of turbulence and is useful in better understanding its mechanism. To clarify the dynamo mechanism in the MHD turbulent flow, we examine the transport equations for the turbulent kinetic and magnetic energies. The equations for the two components of the turbulent kinetic energy, $\langle \tilde{u}_x^m \rangle$ and $\langle \tilde{u}_y^m \rangle$, are given by

$$\frac{\partial}{\partial t}\langle \tilde{u}_x^m \rangle = P_{Ks,K} + P_{Ks,M} + L_{Ks} + \pi_{Ks} - e_{Ks} + D_{Ks},$$

$$P_{Ks,K} = -2 \langle \tilde{u}_x^m \tilde{u}_x^m \rangle \frac{\partial U_x}{\partial y}, \quad P_{Ks,M} = 2 \langle \tilde{u}_y^m \tilde{b}_y^m \rangle \frac{\partial B_x}{\partial y},$$

$$L_{Ks} = 2 \left\langle \tilde{u}_x^m \frac{\partial \tilde{b}_x^m}{\partial x} (B_k + \tilde{b}_k) \right\rangle, \quad \pi_{Ks} = 2 \left\langle \tilde{p}_M \frac{\partial \tilde{u}_y^m}{\partial x} \right\rangle,$$

$$\frac{\partial}{\partial t}\langle \tilde{u}_y^m \rangle = L_{Ks} + \pi_{Ks} - e_{Ks} + D_{Ks},$$

$$L_{Ks} = 2 \left\langle \tilde{u}_x^m \frac{\partial \tilde{b}_x^m}{\partial x} (B_k + \tilde{b}_k) \right\rangle, \quad \pi_{Ks} = 2 \left\langle \tilde{p}_M \frac{\partial \tilde{u}_x^m}{\partial x} \right\rangle.$$
and magnetic energies, and $\Pi_{K_x}$ and $\Pi_{K_y}$ are the pressure-strain terms that lead to the energy redistribution between the three components of the turbulent kinetic energy. Detailed expressions for the dissipation terms $\epsilon_{K_x}$, $\epsilon_{K_y}$ and the diffusion terms $D_{K_x}$, $D_{K_y}$ are omitted here. The production term $P_{K_{xK}}$ involving $\partial U_i/\partial y$ represents the energy transfer from $U_i^2$ to $\langle u_i^2 \rangle$, whereas the production term $P_{K_{xM}}$ involving $\partial B_i/\partial y$ stands for the energy transfer from $B_i^2$ to $\langle u_i^2 \rangle$. There is no production term in the $\langle u_i^2 \rangle$ equation.

Figure 13(a) shows profiles of the six terms on the right-hand side of Eq. (32) for $\langle u_i^2 \rangle$. Like those in the non-MHD case, the production term $P_{K_{xK}}$ involving $\partial U_i/\partial y$ and the dissipation term $\epsilon_{K_i}$ are dominant as the energy gain and loss, respectively. The pressure-strain term shows small negative values, which represent the energy redistribution from $\langle u_i^2 \rangle$ to the other components of the turbulent kinetic energy. The magnitude of the pressure-strain term relative to the dissipation term is small compared to the non-MHD case; the small value of the energy redistribution accounts for the strong anisotropy of the turbulent kinetic energy, as shown in Fig. 5. The production term $P_{K_{xM}}$ involving $\partial B_i/\partial y$ and the Lorentz-force term $L_{K_i}$ appear only in the MHD case. We should note that the production term $P_{K_{xM}}$ shows negative values. The negative value means that the energy is transferred from $\langle u_i^2 \rangle$ to $B_i^2$; this transfer indeed represents the dynamo effect. Since the production term $P_{K_{xM}}$ involves the correlation $\langle u_i u_i \rangle$, the magnetic-field fluctuation $\overline{B_i}$ is necessary for this energy transfer. The transport equation for the turbulent magnetic energy will be discussed later.

Figure 13(b) shows profiles of the four terms on the right-hand side of Eq. (35) for $\langle u_i^2 \rangle$. Since there is no production term, the pressure-strain term acts as the energy gain; the energy is redistributed from $\langle u_i^2 \rangle$ to $\langle u_i^2 \rangle$. Both the dissipation and Lorentz-force terms show negative values. The Lorentz-force term represents the energy transfer from $\langle u_i^2 \rangle$ to $\langle \overline{B_i^2} \rangle$ as will soon be shown.

Next, we examine the transport equations for the turbulent magnetic energy. The equations for the two components of the turbulent magnetic energy, $\langle B_x'^2 \rangle$ and $\langle B_y'^2 \rangle$, are given by

$$\frac{\partial}{\partial t} \langle \overline{B_x'^2} \rangle = P_{M_{xK}} + P_{M_{xM}} + L_{M_x} - \epsilon_{M_x} + D_{M_x},$$

$$P_{M_{xK}} = 2\langle \overline{B_x'\overline{B_x'}} \rangle \frac{\partial U_x}{\partial y}, \quad P_{M_{xM}} = -2\langle \overline{B_x'\overline{B_x'}} \rangle \frac{\partial B_x}{\partial y},$$

$$L_{M_x} = 2\left( \overline{\partial u_x'^2} \delta \overline{B_x} - \overline{\partial u_x'^2} B_x + \overline{\partial u_y'^2} \right),$$

$$L_{M_y} = 2\left( \overline{\partial u_y'^2} \delta \overline{B_y} - \overline{\partial u_y'^2} B_y + \overline{\partial u_x'^2} \right).$$

Here, $P_{M_{xK}}$ and $P_{M_{xM}}$ are the production terms related to the mean field, whereas $L_{M_x}$ and $L_{M_y}$ are the Lorentz-force terms that represent the energy transfer between turbulent kinetic and magnetic energies. Detailed expressions for the dissipation terms $\epsilon_{M_x}$, $\epsilon_{M_y}$ and the diffusion terms $D_{M_x}$, $D_{M_y}$ are omitted here. In these equations there is no pressure-strain term because the pressure gradient term does not appear in the induction equation for the magnetic field. Thus, there is no energy redistribution between the three components of the turbulent magnetic energy.

Figure 14(a) shows profiles of the five terms on the right-hand side of Eq. (37) for $\langle B_x'^2 \rangle$. Like the $\langle u_i^2 \rangle$ equation the production term $P_{M_{xK}}$ involving $\partial U_i/\partial y$ and the dissipation term $\epsilon_{M_i}$ are dominant. In this case the production term $P_{M_{xM}}$ involving $\partial B_i/\partial y$ shows positive values, which represent the energy transfer from $B_i^2$ to $\langle \overline{B_i^2} \rangle$, in contrast to the negative production term $P_{K_{xM}}$ for $\langle u_i^2 \rangle$. The difference in the direction of the energy transfer is related to the difference in the sign between $\langle \overline{u_i u_i} \rangle$ and $-\langle \overline{u_i u_i} \rangle$ plotted in Fig. 7. The positive gradient of $\langle \overline{B_i^2} \rangle$ in Fig. 7 means the increase of $B_i$ in the mean magnetic-field equation and leads to the negative production term $P_{K_{xM}}$ in the $\langle u_i^2 \rangle$ equation in Fig. 13(a). On the other hand, the negative gradient of $-\langle \overline{u_i u_i} \rangle$ in Fig. 7
FIG. 14. Balance of terms in the transport equations for the turbulent magnetic energy given by Eqs. (37) and (40): (a) streamwise component $\langle \vec{b}^2 \rangle_x$ and (b) wall-normal component $\langle \vec{b}^2 \rangle_z$. The SGS contributions are included in the dissipation and diffusion terms.

means the decrease in $B_z$ in the mean magnetic-field equation and leads to the positive production term $P_{MSM}$ in the $\langle \vec{b}^2 \rangle_z$ equation in Fig. 14(a).

Figure 14(b) shows profiles of the three terms on the right-hand side of Eq. (40) for $\langle \vec{b}^2 \rangle_z$. Instead of the pressure-strain term, the Lorentz-force term acts as the energy gain. It is shown that the magnetic energy $\langle \vec{b}^2 \rangle_z$ is sustained by the energy transfer from the kinetic energy $\langle \vec{u}^2 \rangle_z$. The Lorentz-force term is necessary to generate the magnetic fluctuation $\vec{b}^2_z$ that is required for the dynamo effect as mentioned before.

Figure 15 summarizes the energy transfer with respect to the turbulent kinetic and magnetic energies. As in the case of the non-MHD channel flow, the constant pressure gradient $-\partial p_\parallel / \partial x$ drives the mean velocity $U_x$. The streamwise component of the turbulent kinetic energy $\langle \vec{u}^2 \rangle_x$ is first generated due to the mean-shear production term. The energy is then redistributed to the wall-normal and spanwise components $\langle \vec{u}^2 \rangle_y$ and $\langle \vec{u}^2 \rangle_z$ via the pressure-strain term. In addition to the above process, the energy is also transferred to the magnetic field. Like the kinetic energy $\langle \vec{u}^2 \rangle_z$, the streamwise component of the turbulent magnetic energy $\langle \vec{b}^2 \rangle_z$ is generated due to the mean-shear production term; the energy is transferred from $U_x^2$ to $\langle \vec{b}^2 \rangle_z$. Unlike the kinetic energy, there is no pressure-strain term for the magnetic energy. Instead, the energy is transferred from $\langle \vec{u}^2 \rangle_z$ to $\langle \vec{b}^2 \rangle_z$ via the Lorentz-force term; the same holds for $\langle \vec{b}^2 \rangle_z$. Since there is no mean-field electromotive force $\mathbf{U} \times \mathbf{B}$, the energy is not directly transferred from $U_x^2$ to $B_z^2$. It was shown that the energy is transferred from $\langle \vec{u}^2 \rangle_z$ to $\langle \vec{b}^2 \rangle_z$; this transfer represents the dynamo effect.

IV. CONCLUSIONS

A LES of MHD turbulent channel flow was carried out to investigate the dynamo mechanism in MHD turbulence. We used the SGS model derived by extending the Smagorinsky model to MHD flows. The time evolution of the turbulent field showed that the streamwise component of the mean magnetic field was generated and sustained. We examined the turbulent statistics obtained by averaging over the plane and in time. The balance of the electromotive force showed that the mean magnetic field is generated due to the turbulent electromotive force. We investigate the turbulent electromotive force using its turbulence model to show that the cross-helicity dynamo is important compared to the $\alpha$ term.
dynamo in the present MHD flow. To verify the importance of the cross-helicity dynamo, we examined the transport equation for the turbulent electromotive force; it was actually generated due to the production term related to the cross-helicity dynamo. The transport equations for the turbulent kinetic and magnetic energies were also examined; the dynamo action is shown as the energy transfer from the turbulent kinetic energy to the mean magnetic field.

In this work, the constant pressure gradient was applied to drive the mean flow and turbulence. The initial mean magnetic field was set to zero by considering the electromotive force. As a result, the mean magnetic field was generated due to the cross-helicity dynamo. However, this result does not mean that other dynamo mechanisms are irrelevant, in general. They may be important in other type of MHD turbulence; such MHD flows can be simulated by imposing different conditions. For example, instead of the pressure gradient, the heat flux or the external electric field can produce MHD turbulence. The system rotation must be important for the \( \alpha \) dynamo effect. Nonzero initial mean magnetic field can also be adopted. We expect that the LES of MHD turbulent flows under various conditions is useful for better understanding the dynamo mechanism in MHD turbulence and for improving the turbulence model.

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APPENDIX: SGS MODEL FOR MHD TURBULENCE

Using the TSDIA, Yoshizawa\(^{17}\) derived a SGS model for MHD turbulence. In his model, the SGS stress is given by the eddy-viscosity model and the SGS electromotive force is expressed in terms of the eddy-diffusivity term and the pumping-effect term. Hamba\(^{18}\) modified the SGS model so that the \( \alpha \) dynamo term is involved instead of the pumping-effect term. In this work we use the SGS model that involves only the eddy-viscosity and eddy-diffusivity terms such as the Smagorinsky model.

In this appendix we explain the derivation of the SGS model. The SGS stress and electromotive force are assumed to be given by Eqs. (8) and (9) in Sec. II, respectively. We then need to model the SGS viscosity and diffusivity. As fundamental quantities we adopt the filter width \( \Delta \) and the dissipation rate \( \varepsilon \) of the turbulent MHD energy \( \left[ \overline{u_i^2} + \overline{b_i^2} \right] \) \(-\left[ \overline{\bar{u}_i^2} + \overline{\bar{b}_i^2} \right]\)/2. The latter choice is because this dissipation rate is considered as the energy cascade rate for the energy spectrum in the MHD turbulence theory.\(^{34-36}\) By dimensional analysis the SGS viscosity and diffusivity can be written as

\[
\nu_{\text{SGS}} = C_v \varepsilon_{\text{SGS}}^{1/3} \Delta^{2/3}, \quad \lambda_{\text{SGS}} = C_\lambda \varepsilon_{\text{SGS}}^{1/3} \Delta^{2/3},
\]

(A1)

respectively, where \( C_v \) and \( C_\lambda \) are nondimensional constants.

As is the case of the Smagorinsky model, we assume the local equilibrium; that is, the production term \( P_{\text{SGS}} \) and the dissipation term \( \varepsilon_{\text{SGS}} \) balance each other in the transport equation for the turbulent MHD energy as follows:

\[
P_{\text{SGS}} = -\tau_{ij} \frac{\partial \bar{u}_i}{\partial x_j} - \epsilon_{\text{M}} \cdot \mathbf{j} = \varepsilon_{\text{SGS}}.
\]

(A2)

Substituting Eqs. (8) and (9) into Eq. (A2) yields

\[
\frac{1}{2} \nu_{\text{SGS}} \overline{\bar{u}_i^2} + \lambda_{\text{SGS}} \overline{\bar{b}_i^2} = \varepsilon_{\text{SGS}}.
\]

(A3)

Substituting Eq. (A1) into Eq. (A3) leads to the following expression for the dissipation rate:

\[
\varepsilon_{\text{SGS}} = \left( \frac{1}{2} C_v \overline{\bar{u}_i^2} + C_\lambda \overline{\bar{b}_i^2} \right)^{3/2} \Delta^2.
\]

(A4)

Substituting this expression into Eq. (A1), we obtain Eqs. (10) and (11) for the SGS viscosity and diffusivity.

Using the TSDIA, the ratio of the turbulent viscosity \( \nu_T \) to the turbulent diffusivity \( \beta \) can be derived theoretically as

\[
\frac{\nu_T}{\beta} = \frac{7}{5}.
\]

(A5)

In this work, we adopt this ratio for the SGS viscosity and diffusivity: \( \nu_{\text{SGS}} / \lambda_{\text{SGS}} = 7/5 \). This value leads to \( C_v / C_\lambda = 7/5 \) as shown in Eq. (12).

Finally, we assume that in the non-MHD limit the present model should be reduced to the Smagorinsky model with the Smagorinsky constant \( C_S = 0.1 \). In the case of \( \mathbf{b} = 0 \), the SGS viscosity can be written as

\[
\nu_{\text{SGS}} = (C_v^{3/4} \Delta)^2 \left( \frac{1}{2} \overline{\bar{u}_i^2} \right)^{1/2},
\]

(A6)

which is the same form as the Smagorinsky model. Since the model constant \( C_v \) is related to \( C_S \) as follows:

\[
C_v^{3/4} = C_S = 0.1,
\]

(A7)

we obtain the value \( C_v = 0.046 \), as shown in Eq. (12).

21 W.-C. Müller and D. Carati, Phys. Plasmas 9, 824 (2002).